

# Finding resource states of measurement-based quantum computing is harder than quantum computing

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Measurement-based quantum computing enables universal quantum computing with only adaptive single-qubit measurements on certain many-qubit states, such as the graph state, the Affleck-Kennedy-Lieb-Tasaki (AKLT) state, and several tensor-network states. Finding new resource states of measurement-based quantum computing is a hard task, since for a given state there are exponentially many possible measurement patterns on the state. In this paper, we consider the problem of deciding, for a given state and a set of unitary operators, whether there exists a way of measurement-based quantum computing on the state that can realize all unitaries in the set, or not. We show that the decision problem is QCMA-hard, which means that finding new resource states of measurement-based quantum computing is harder than quantum computing itself (unless BQP is equal to QCMA). We also derive an upperbound of the decision problem: the problem is in a quantum version of the second level of the polynomial hierarchy.

Measurement-based quantum computing [1] is another model of quantum computing than the traditional circuit model where universal quantum computing can be done with only adaptive single-qubit measurements on certain many-qubit states which are called resource states. Although it is mathematically equivalent to the circuit model, the clear separation between the resource preparation phase and the resource consumption phase has enabled plenty of new results in, for example, fault-tolerant quantum computing [2], condensed matter physics [3–8], studying roles of quantumness in quantum computing [9, 10], secure quantum computing (blind quantum computing) [11, 12], and quantum complexity theory [13–15].

The first and the most standard example of universal resource states is the graphs state [1], which is obtained by applying  $CZ$  operators on all connected  $|+\rangle$  states placed on every vertex of a graph. Researchers have tried to find more condensed-matter physically motivated resource states. For example, the Affleck-Kennedy-Lieb-Tasaki (AKLT) state [16] was found to be a universal resource state [3–6]. Several tensor-network states were also shown to be universal resource states by considering virtual quantum computing in the correlation space [17]. Furthermore, low-temperature thermal equilibrium states of some physically motivated Hamiltonians were shown to be universal resource states for topological measurement-based quantum computing [18, 19]. In spite of much efforts, however, we have only a very short list of universal resource states. One of the main reasons of the difficulty of finding new resource states is the exponential increase of possible measurement patterns on a given state. Therefore we have a natural question: how hard is it to find a new resource state? Is it, say, NP-hard?

In this paper, we study the computational complexity of finding new resource states of measurement-based quantum computing. We consider the problem of deciding, for a given state and a set of unitary operators,

whether there exists a way of measurement-based quantum computing on the state that can realize all unitaries in the set, or not. We show that the decision problem is QCMA-hard. The class QCMA [20, 21] is a quantum version of NP and defined in the following way:

A language  $L$  is in QCMA if and only if there exists a uniformly-generated family  $\{V_x\}_x$  of polynomial-size quantum circuits such that

- If  $x \in L$ , then there exists a  $w$ -bit string  $y \in \{0, 1\}^w$  such that the probability of obtaining 1 when the first qubit of  $V_x(|y\rangle \otimes |0^n\rangle)$  is measured in the computational basis is  $\geq \frac{2}{3}$ . Here,  $n = \text{poly}(|x|)$  and  $w = \text{poly}(|x|)$ .
- If  $x \notin L$ , then for any  $w$ -bit string  $y \in \{0, 1\}^w$ , the probability is  $\leq \frac{1}{3}$ .

It is known that the error bound  $(\frac{2}{3}, \frac{1}{3})$  can be amplified to  $(1 - 2^{-r}, 2^{-r})$  for any polynomial  $r$  by using the standard argument of the error reduction used in other probabilistic classes such as BPP, MA, and BQP. Obviously QCMA contains BQP. (We have only to ignore the witness.) Moreover, QCMA seems to be strictly larger than BQP, since it seems to be difficult to find a correct  $y$  in a quantum polynomial time. In fact, there are several results that support  $\text{QCMA} \neq \text{BQP}$ . (For example, it is obvious that QCMA contains NP. However, BQP is not believed to contain NP [22].) Therefore, if we assume  $\text{QCMA} \neq \text{BQP}$ , we can put our result concisely as follows: “finding new resource states is harder than quantum computing itself”.

We also study upperbounds of the problem. We show that the problem is in a quantum version of the second level of the polynomial hierarchy. The polynomial hierarchy is one of the most important concepts in complexity theory, and its quantum versions were considered in Refs. [23, 24].

*Measurement-based quantum computing.*— Before giving the precise definition of the problem that we show to

be QCMA-hard, let us here explain an abstract form of measurement-based quantum computing. Assume that as a resource state, we are given an  $N$ -qubit state  $|\Psi\rangle$ . Let  $\mathcal{U} \equiv \{U_y\}_{y \in \{0,1\}^w}$  be a set of unitary operators acting on  $n$  qubits ( $n \leq N$ ). We say that the resource state  $|\Psi\rangle$  is  $\mathcal{U}$ -universal with precision  $\epsilon$  ( $0 \leq \epsilon \leq 1$ ) if there exists a polynomial-time classical algorithm  $\Lambda$  such that for any  $y \in \{0,1\}^w$ ,

1. We input  $(1, y)$  to  $\Lambda$ .  $\Lambda$  outputs a classical description of a single-qubit unitary operator  $u_1$ . We measure the first qubit of the resource state in the basis  $\{u_1|0\rangle, u_1|1\rangle\}$ . We obtain the measurement result  $m_1 \in \{0,1\}$ .
2. We input  $(2, y, m_1)$  to  $\Lambda$ .  $\Lambda$  outputs a classical description of a single-qubit unitary operator  $u_2$ . We measure the second qubit of the resource state in the basis  $\{u_2|0\rangle, u_2|1\rangle\}$ . We obtain the measurement result  $m_2 \in \{0,1\}$ .
3. We input  $(3, y, m_1, m_2)$  to  $\Lambda$ .  $\Lambda$  outputs a classical description of a single-qubit unitary operator  $u_3$ . We measure the third qubit of the resource state in the basis  $\{u_3|0\rangle, u_3|1\rangle\}$ . We obtain the measurement result  $m_3 \in \{0,1\}$ .
4. In this way, we repeat this adaptive single-qubit measurements until all but the last  $n$  qubits of the resource state are measured. In other words, when we measure  $j$ th qubit of the resource state, we input  $(j, y, m_1, \dots, m_{j-1})$  to  $\Lambda$ , and get a classical description of a single-qubit unitary operator  $u_j$  from  $\Lambda$ . We then measure  $j$ th qubit of the resource state in the basis  $\{u_j|0\rangle, u_j|1\rangle\}$ , and obtain the measurement result  $m_j$ . We repeat it until  $j = N - n$ . Let  $|\psi'_m\rangle$ , where  $m \equiv (m_1, \dots, m_{N-n})$ , be the (normalized) post-measurement state of  $n$  qubits of the resource state that are not measured. We also denote the probability of obtaining  $m$  by  $p_m$ . (For example, if  $|\Psi\rangle$  is the graph state,  $p_m = 2^{-(N-n)}$  for all  $m$ , and  $|\psi'_m\rangle$  is equal to  $U_y|0^n\rangle$  up to some Pauli byproduct operators.)
5. We input  $(N - n + 1, y, m)$  to  $\Lambda$ .  $\Lambda$  outputs classical descriptions of single-qubit unitary operators  $\{v_j\}_{j=1}^n$ . We apply  $v_j$  on  $j$ th qubit of  $|\psi'_m\rangle$  to obtain  $|\psi_m\rangle \equiv \bigotimes_{j=1}^n v_j |\psi'_m\rangle$ . (This process is the “final byproduct correction”. For example, if  $|\Psi\rangle$  is the graph state, each  $v_j$  is a Pauli byproduct operator, and  $|\psi_m\rangle = U_y|0^n\rangle$  for all  $m$ .)
6. The state  $\sum_m p_m |\psi_m\rangle \langle \psi_m|$  is close to the ideal state  $U_y|0^n\rangle$  in the sense of

$$\frac{1}{2} \left\| \sum_m p_m |\psi_m\rangle \langle \psi_m| - U_y|0^n\rangle \langle 0^n| U_y^\dagger \right\|_1 \leq \epsilon.$$

Here,  $\|X\|_1 \equiv \text{Tr} \sqrt{X^\dagger X}$  is the trace norm.

*The problem.*— Now we define the decision problem that we study, which is a promise version of deciding whether a given state is non- $\mathcal{U}$ -universal or not. We call the problem NONUNIVERSALITY $_\epsilon$  for a parameter  $\epsilon$  ( $0 \leq \epsilon \leq 1$ ):

- Input:  $\mathcal{U}$  and  $|\Psi\rangle$ .
- YES:  $|\Psi\rangle$  is not  $\mathcal{U}$ -universal. In other words, for any  $\Lambda$  there exists  $y$  such that,

$$\frac{1}{2} \left\| \sum_m p_m |\psi_m\rangle \langle \psi_m| - U_y|0^n\rangle \langle 0^n| U_y^\dagger \right\|_1 \geq 1 - \epsilon.$$

- NO:  $|\Psi\rangle$  is  $\mathcal{U}$ -universal. In other words, there exists  $\Lambda$  such that for all  $y$

$$\frac{1}{2} \left\| \sum_m p_m |\psi_m\rangle \langle \psi_m| - U_y|0^n\rangle \langle 0^n| U_y^\dagger \right\|_1 \leq \epsilon.$$

The main result of the present paper is that the problem is QCMA-hard for  $\epsilon = 2^{-t}$ , where  $t$  is any polynomial.

*Proof.*— Here we give a proof. Let us assume that a language  $L$  is in QCMA, and let  $V_x$  be the corresponding verification circuit for an instance  $x$ . Due to the standard argument of the error reduction, we can assume without loss of generality that the acceptance probability  $p$  satisfies  $p \geq 1 - 2^{-r}$  if  $x \in L$  and  $p \leq 2^{-r}$  if  $x \notin L$ , where  $r$  is any polynomial. Fix one  $x$ . From  $V_x$ , we construct the unitary operator  $U$  that acts on  $n + w + 2r + 1$  qubits as follows (see Fig. 1):

1. Apply  $V_x$  on  $|y0^n\rangle$  to generate  $V_x|y0^n\rangle$ .
2. Add an ancilla qubit initialized in  $|0\rangle_1$  to generate  $V_x|y0^n\rangle \otimes |0\rangle_1$ .
3. Flip the ancilla qubit if and only if the first qubit of  $V_x|y0^n\rangle$  is  $|1\rangle$ . We therefore obtain

$$\begin{aligned} & (|0\rangle\langle 0| \otimes I^{\otimes n+w-1}) V_x|y0^n\rangle |0\rangle_1 \\ & + (|1\rangle\langle 1| \otimes I^{\otimes n+w-1}) V_x|y0^n\rangle |1\rangle_1. \end{aligned}$$

Here  $I \equiv |0\rangle\langle 0| + |1\rangle\langle 1|$  is the two-dimensional identity operator.

4. Add  $2r$  ancilla qubits initialized in  $|0^{2r}\rangle_2$ . We obtain

$$\begin{aligned} & (|0\rangle\langle 0| \otimes I^{\otimes n+w-1}) V_x|y0^n\rangle |0\rangle_1 |0^{2r}\rangle_2 \\ & + (|1\rangle\langle 1| \otimes I^{\otimes n+w-1}) V_x|y0^n\rangle |1\rangle_1 |0^{2r}\rangle_2. \end{aligned}$$

5. Apply a  $2r$ -qubit unitary operator  $ME$  on the  $2r$  ancilla qubits  $|0^{2r}\rangle_2$  that changes the state  $|0^{2r}\rangle$  to the maximally-entangled state

$$|ME\rangle \equiv \frac{1}{\sqrt{2^r}} \sum_{j=1}^{2^r} |j\rangle |j\rangle$$

if and only the first qubit of  $V_x|y0^n\rangle$  is  $|1\rangle$ . We therefore obtain

$$\begin{aligned} & (|0\rangle\langle 0| \otimes I^{\otimes n+w-1})V_x|y0^n\rangle|0\rangle_1|0^{2r}\rangle_2 \\ & + (|1\rangle\langle 1| \otimes I^{\otimes n+w-1})V_x|y0^n\rangle|1\rangle_1|ME\rangle_2. \end{aligned}$$

6. Apply  $V_x^\dagger$  on the main register. We thus obtain the final state

$$\begin{aligned} & U|y0^n\rangle|0\rangle_1|0^{2r}\rangle_2 \\ & = V_x^\dagger(|0\rangle\langle 0| \otimes I^{\otimes n+w-1})V_x|y0^n\rangle|0\rangle_1|0^{2r}\rangle_2 \\ & + V_x^\dagger(|1\rangle\langle 1| \otimes I^{\otimes n+w-1})V_x|y0^n\rangle|1\rangle_1|ME\rangle_2. \end{aligned}$$

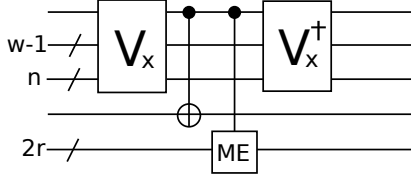


FIG. 1: The unitary operator  $U$ . ME means the unitary operator that changes  $|0^{2r}\rangle$  to the maximally entangled state  $|ME\rangle$ .

Let us define

$$U_y \equiv U \left[ \left( \bigotimes_{j=1}^w X_j^{y_j} \right) \otimes I^{\otimes n+2r+1} \right],$$

where  $y_j$  is the  $j$ th bit of  $y$ , and  $X_j \equiv |0\rangle\langle 1| + |1\rangle\langle 0|$  is the bit-flip operator acting on  $j$ th qubit. We also define

$$|\Psi\rangle \equiv |0^{n+w+2r+2}\rangle.$$

First, we consider the case of  $x \in L$ . In this case,  $p \geq 1 - 2^{-r}$  for a certain  $y$ . Note that whatever  $\Lambda$  we choose, what we can do is just measuring a single qubit of  $|\Psi\rangle$  and then rotating each of the  $n + w + 2r + 1$  unmeasured qubits. Therefore, for any  $\Lambda$ ,  $|\psi_m\rangle$  is an  $(n + w + 2r + 1)$ -qubit product state for all  $m$ . Therefore, from Uhlmann's theorem,

$$\begin{aligned} & |\langle \psi_m | U_y | 0^{n+w+2r+1} \rangle|^2 \\ & = |\langle \psi_m | U | y 0^{n+2r+1} \rangle|^2 \\ & \leq F(\rho, \sigma)^2 \\ & = (1-p) |\langle \xi_1 \xi_2 | 0^{2r} \rangle|^2 + p |\langle \xi_1 \xi_2 | ME \rangle|^2 \\ & \leq (1-p) + p |\langle \xi_1 \xi_2 | ME \rangle|^2 \\ & \leq (1-p) + p F \left( |\xi_1\rangle\langle \xi_1|, \frac{I^{\otimes r}}{2^r} \right)^2 \\ & = 1 - p + \frac{p}{2^r} \\ & \leq 2^{-r} + 2^{-r} \\ & = 2^{-r+1}, \end{aligned}$$

where  $F(\rho, \sigma) \equiv \text{Tr} \sqrt{\sqrt{\rho} \sigma \sqrt{\rho}}$  is the fidelity,

$$\begin{aligned} \rho & \equiv \text{Tr}_2(|\psi_m\rangle\langle \psi_m|) \\ & \equiv |\xi_1\rangle\langle \xi_1| \otimes |\xi_2\rangle\langle \xi_2|, \\ \sigma & \equiv \text{Tr}_2(U|y0^{n+2r+1}\rangle\langle y0^{n+2r+1}|U^\dagger) \\ & = (1-p)|0^{2r}\rangle\langle 0^{2r}| + p|ME\rangle\langle ME|, \end{aligned}$$

$\text{Tr}_2$  is the partial trace except for the send ancilla register, and  $|\xi_j\rangle$  ( $j = 1, 2$ ) is a certain (actually product)  $r$ -qubit state. Therefore

$$\begin{aligned} & F \left( \sum_m p_m |\psi_m\rangle\langle \psi_m|, U_y | 0^{n+w+2r+1} \rangle \langle 0^{n+w+2r+1} | U_y^\dagger \right) \\ & = \sqrt{\sum_m p_m |\langle \psi_m | U_y | 0^{n+w+2r+1} \rangle|^2} \\ & \leq 2^{\frac{-r+1}{2}}. \end{aligned}$$

Hence we have shown that for any  $\Lambda$ , there exists  $y$  such that

$$\begin{aligned} & \frac{1}{2} \left\| \sum_m p_m |\psi_m\rangle\langle \psi_m| - U_y | 0^{n+w+2r+1} \rangle \langle 0^{n+w+2r+1} | U_y^\dagger \right\|_1 \\ & \geq 1 - 2^{\frac{-r+1}{2}} \\ & \geq 1 - 2^{-t}, \end{aligned}$$

where we have taken  $r \geq 2t + 1$ .

Next, we consider the case of  $x \notin L$ . In this case,  $p \leq 2^{-r}$  for any  $y$ . We define  $\Lambda$  in such a way that

$$|\psi_m\rangle = |y0^{n+2r+1}\rangle$$

for any  $m$ . This is trivially possible as follows:

1. Measure the first qubit of  $|\Psi\rangle = |0^{n+w+2r+2}\rangle$  in the computational basis. Then we obtain  $|\tilde{\psi}_{m=0}\rangle = |0^{n+w+2r+1}\rangle$  with probability 1.
2. Apply  $X_j^{y_j}$  on the  $j$ th qubit of  $|\tilde{\psi}_{m=0}\rangle$  for  $j = 1, \dots, w$  to obtain  $|\psi_{m=0}\rangle = |y0^{n+2r+1}\rangle$ .

Then, for any  $y$ ,

$$\begin{aligned} & \sum_m p_m |\langle \psi_m | U_y | 0^{n+w+2r+1} \rangle|^2 \\ & = |\langle \psi_{m=0} | U_y | 0^{n+w+2r+1} \rangle|^2 \\ & = |\langle y 0^{n+2r+1} | U | y 0^{n+2r+1} \rangle|^2 \\ & = |\langle y 0^n | V_x^\dagger (|0\rangle\langle 0| \otimes I^{\otimes n+w-1}) V_x | y 0^n \rangle|^2 \\ & = (1-p)^2 \\ & \geq (1 - 2^{-r})^2 \\ & = 1 - 2^{-r+1} + 2^{-2r} \\ & \geq 1 - 2^{-r+1}. \end{aligned}$$

Therefore

$$\begin{aligned} & F\left(\sum_m p_m |\psi_m\rangle\langle\psi_m|, U_y |0^{n+w+2r+1}\rangle\langle 0^{n+w+2r+1}| U_y^\dagger\right) \\ &= \sqrt{\sum_m p_m |\langle\psi_m| U_y |0^{n+w+2r+1}\rangle|^2} \\ &\geq \sqrt{1 - 2^{-r+1}}. \end{aligned}$$

Hence we have shown that there exists  $\Lambda$  such that for any  $y$

$$\begin{aligned} & \frac{1}{2} \left\| \sum_m p_m |\psi_m\rangle\langle\psi_m| - U_y |0^{n+w+2r+1}\rangle\langle 0^{n+w+2r+1}| U_y^\dagger \right\|_1 \\ &\leq 2^{\frac{-r+1}{2}} \\ &\leq 2^{-t}. \end{aligned}$$

In summary, we have shown that the promise problem is QCMA-hard.

*Upperbound.*— In this paper, we have shown that the problem NONUNIVERSALITY is QCMA-hard. In other words, we have derived an lower bound of the problem. It is an important open problem to find any better lower bound and better upper bound of the problem, or to show that the problem is complete for a complexity class. Here we point out that a quantum version of  $\Pi_2$ , which we call  $\text{Q}\Pi_2$ , is an upper bound of the problem. We define the class  $\text{Q}\Pi_2$  as follows:

A language  $L$  is in  $\text{Q}\Pi_2(a, b)$  if and only if there exists a uniformly generated family  $\{V_x\}_x$  of polynomial-size quantum circuits such that

- If  $x \in L$  then for any  $\lambda$ -bit string  $\Lambda$  there exists a  $w$ -bit string  $y$  such that the probability of obtaining 1 when the first qubit of  $V_x(|\Lambda\rangle|y\rangle|0^n\rangle)$  is measured in the computational basis is  $\geq a$ . Here,  $\lambda, w, n = \text{poly}(|x|)$ .
- If  $x \notin L$  then there exists a  $\lambda$ -bit string  $\Lambda$  such that for any  $w$ -bit string  $y$  the probability is  $\leq b$ .

It is obvious that  $\text{Q}\Pi_2$  is in PSPACE. (We have only to try all possible  $\Lambda$  and  $y$ .) Other types of quantum generalizations of the polynomial hierarchy were studied in Refs. [23, 24].

We can show that the problem NONUNIVERSALITY is in  $\text{Q}\Pi_2(1 - 2\epsilon, 2\epsilon)$ . In fact, since measurement-based quantum computing can be simulated by a circuit model, there exists a polynomial-size quantum circuit  $V$  and polynomials  $t$  and  $n$  such that the reduced density operator of some  $n$  qubits of  $V(|\Lambda\rangle|y\rangle|0^t\rangle)$  is

$$\rho \equiv U_y^\dagger \left( \sum_m p_m |\psi_m\rangle\langle\psi_m| \right) U_y.$$

We then measure all qubits of  $\rho$  in the computational basis, and reject if and only if all qubits are 0. The

acceptance probability  $p$  is therefore

$$p = 1 - \langle 0^n | U_y^\dagger \left( \sum_m p_m |\psi_m\rangle\langle\psi_m| \right) U_y | 0^n \rangle,$$

which means

$$1 - \sqrt{1 - p} \leq \frac{1}{2} \left\| \sum_m p_m |\psi_m\rangle\langle\psi_m| - U_y | 0^n \rangle\langle 0^n | U_y^\dagger \right\|_1 \leq \sqrt{p}.$$

Therefore, for the yes case, for any  $\Lambda$ , there exists  $y$  such that  $1 - \epsilon \leq \sqrt{p}$ , which means

$$\begin{aligned} p &\geq 1 - 2\epsilon + \epsilon^2 \\ &\geq 1 - 2\epsilon. \end{aligned}$$

For the no case, there exists  $\Lambda$  such that for any  $y$ ,  $1 - \sqrt{1 - p} \leq \epsilon$ , which means

$$\begin{aligned} p &\leq 2\epsilon - \epsilon^2 \\ &\leq 2\epsilon. \end{aligned}$$

Hence we have shown that the problem is in  $\text{Q}\Pi_2(1 - 2\epsilon, 2\epsilon)$ .

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